Transmission and Distribution of Electrical Power

By

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Lecture (3)

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Transmission Line Models and Calculations

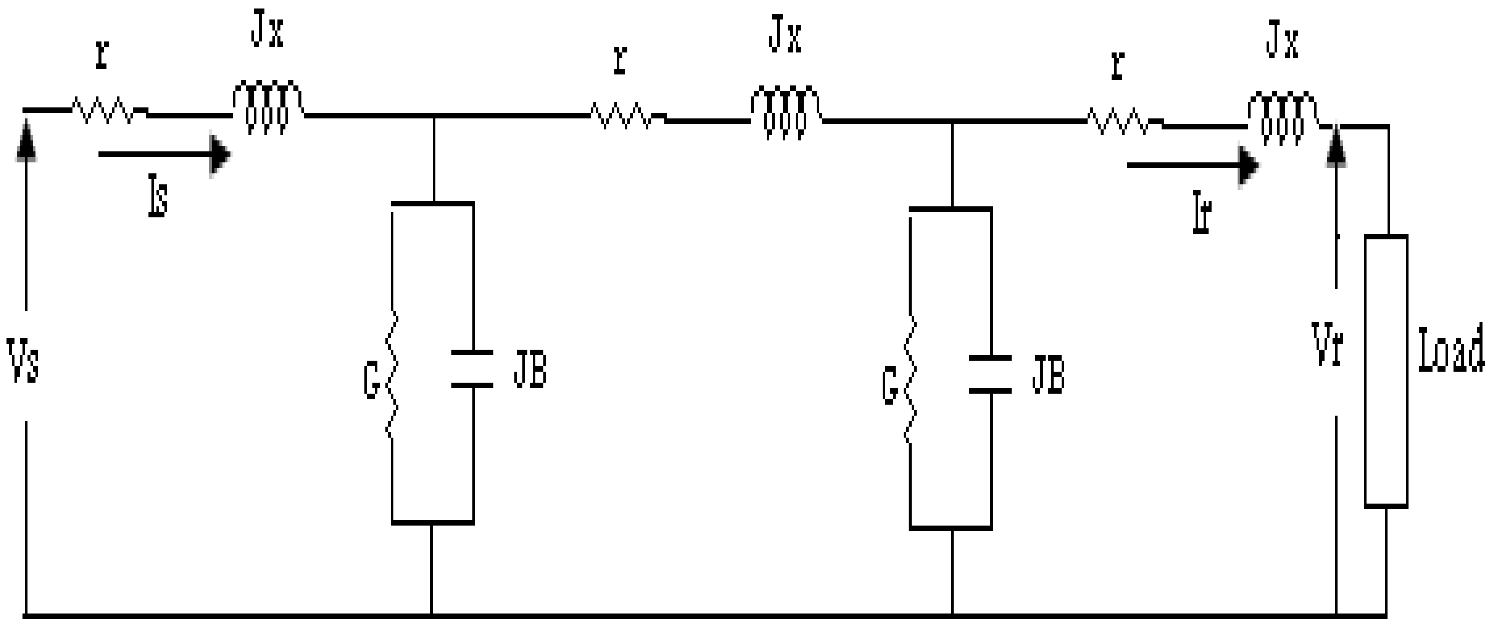
- Classification of transmission lines according to line length:

  - Short transmission line ≤ 80 Km
  - Medium transmission line 80 : 240 Km
  - Long transmission line ≥ 240 Km

Chapter 2:
Transmission Line Models and Calculations

- Classification of transmission lines according to line length:
  - Short transmission line ≤ 80 Km
  - Medium transmission line 80 : 240 Km
  - Long transmission line ≥ 240 Km

Long Transmission Line Representation



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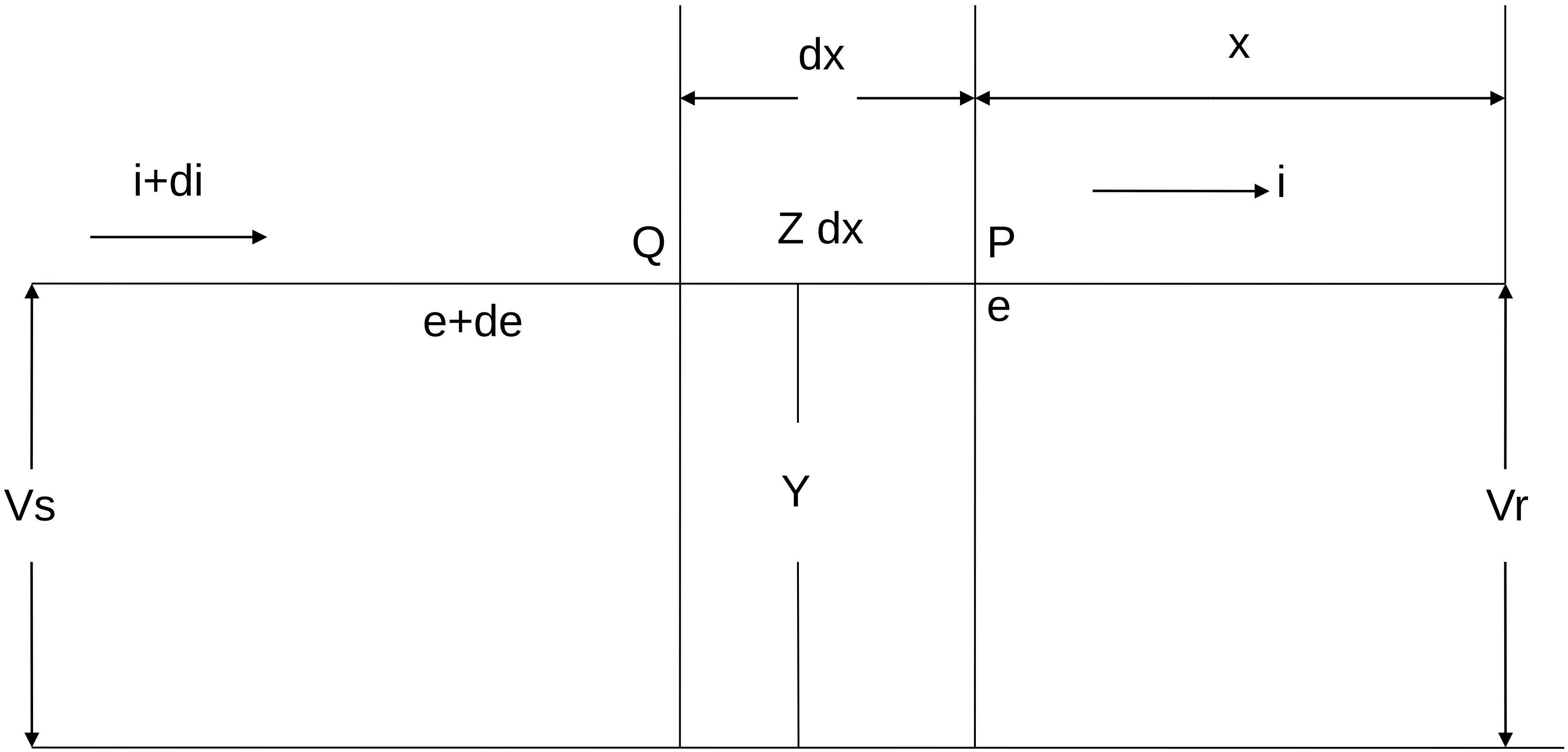

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$$e + de = e + izdx$$

$$\frac{de}{dx} = iz$$

$$also, i + di = i + eydx$$

$$\frac{di}{dx} = ey$$

$$\frac{d^2e}{dx^2} = \frac{di}{dx}z$$

$$= eyz = \alpha^2 e$$

Continue

Where α : Probagation const. $= \sqrt{zy}$

also,

$$\frac{d^2 i}{dx^2} = \frac{de}{dx} y$$

$$= izy$$

$$= \mathrm{i}\alpha^2$$

solving the two second order differential equations yields,

$$e = A\cosh \alpha\alpha + B\sinh \alpha x$$

$$i = C\cosh \alpha\alpha + D\sinh \alpha x$$

Continue

Using the receiving end condition yields,

$$e = V_r \cosh \alpha x + I_r Z_o \sinh \alpha x$$

$$i = \frac{V_r}{Z_o} \sinh \alpha x + I_r \cosh \alpha x$$

Hence,

$$e_s = V_r \cosh \alpha x + I_r \frac{\mathrm{Z}}{\theta} \sinh \alpha x$$

$$i_s = V_r \frac{\mathrm{Y}}{\theta} \sinh \alpha x + I_r \cosh \alpha x$$

Continue

Hence,

$$A = \cosh\theta \qquad B = Z\frac{\sinh\theta}{\theta}$$

$$C = Y\frac{\sinh\theta}{\theta} \qquad D = \cosh\theta$$

$$V_s = A\,V_r + B\,I_r$$

$$I_s = C\,V_r + D\,I_r$$

In matrix form,

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix}$$

Medium T.L Solution by nominal π-Method

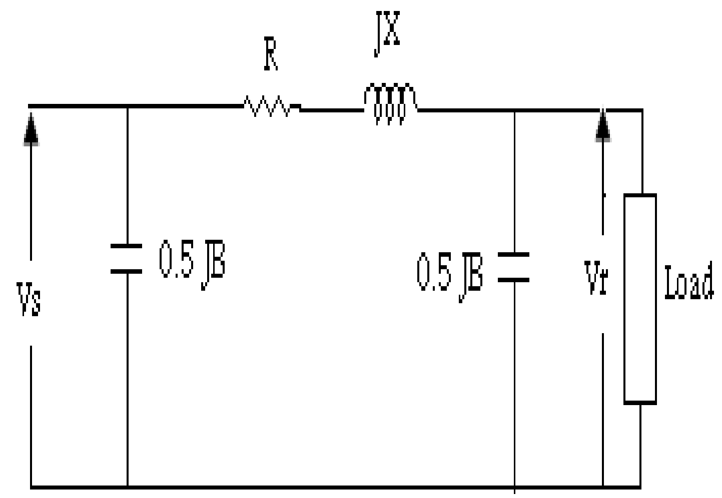
$$V_s = V_r + I_l Z$$

$$I_l = V_r \frac{Y}{2} + I_r$$

$$V_s = V_r + \left(V_r \frac{Y}{2} + I_r \right) Z$$

$$V_s = \left(1 + \frac{YZ}{2} \right) V_r + Z I_r$$

$$I_s = I_l + V_s \frac{Y}{2}$$



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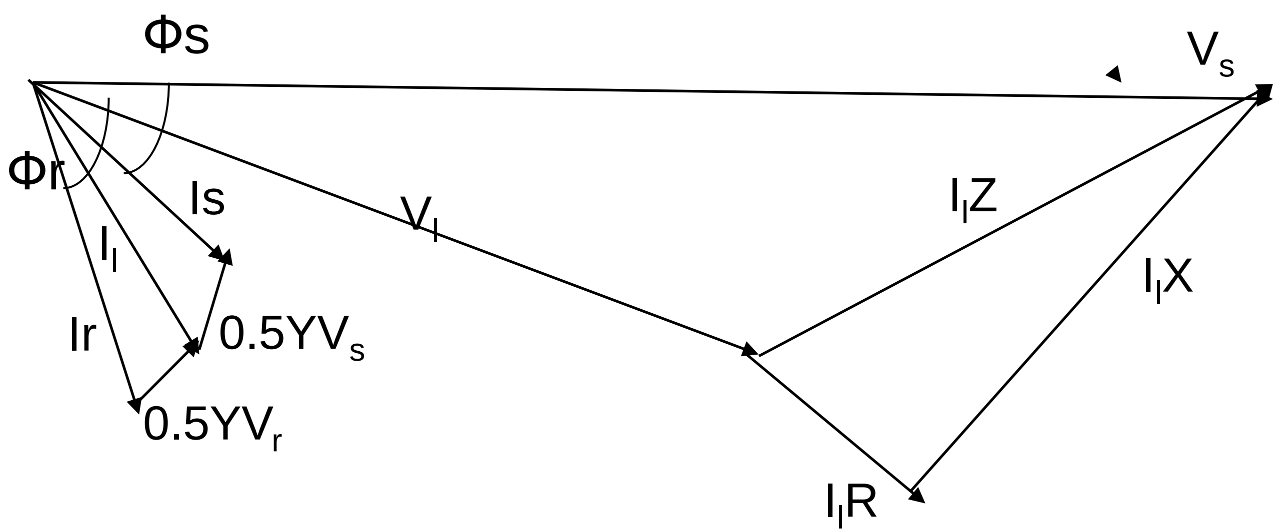

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$$I_s = V_r \frac{Y}{2} + I_r + \frac{Y}{2}\left[\left(1+\frac{YZ}{2}\right)V_r + ZI_r\right]$$

$$= Y\left(1+\frac{YZ}{4}\right)V_r + \left(1+\frac{YZ}{2}\right)I_r$$

$$A = D = 1 + \frac{ZY}{2}$$

$$B = Z$$

$$C = Y\left(1+\frac{YZ}{4}\right)$$

Medium T.L Solution by Nominal T-Method

$$V_s = V_c + I_s \frac{Z}{2}$$

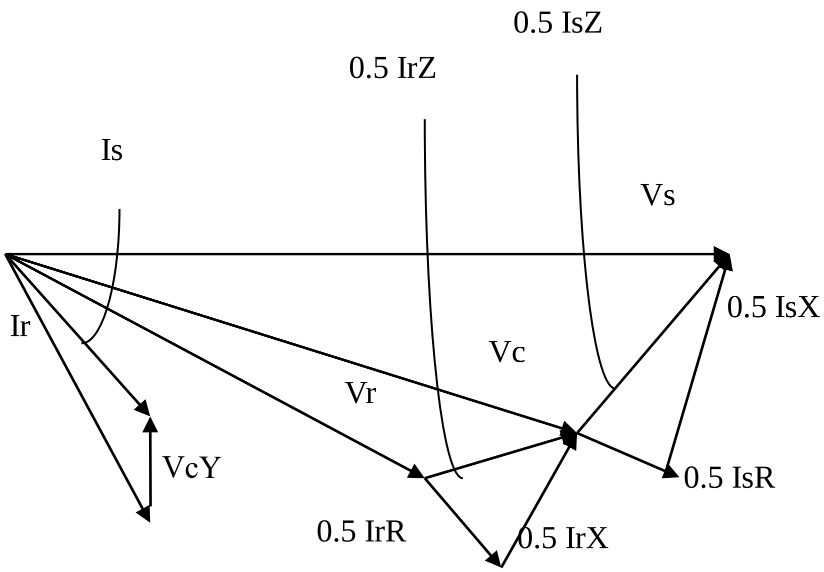
$$V_c = V_r + I_r \frac{Z}{2}$$

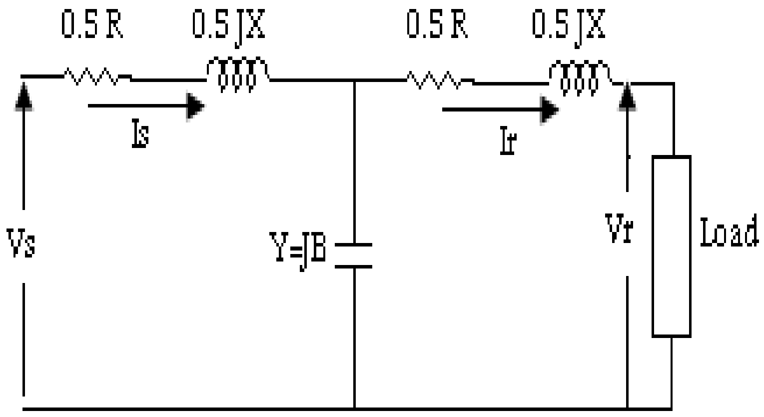
$$I_s = I_c + I_r$$

$$= V_c Y + I_r$$

$$I_s = (V_r + I_r \frac{Z}{2}) y + I_r$$

Continue


0.5 IsZ
0.5 IrZ
Is
Vs
0.5 IsX
Ir
Vc
Vr
VcY
0.5 IsR
0.5 IrR
0.5 IrX


0.5 R
0.5 JX
0.5 R
0.5 JX
Is
Ir
Vs
Y=JB
Vr
Load

Continue

$$I_s = YV_r + \left(1 + \frac{YZ}{2}\right) I_r$$

$$V_s = V_r + I_r \frac{Z}{2} + \frac{Z}{2}\left[YV_r + \left(1 + \frac{YZ}{2}\right) I_r\right]$$

$$V_s = \left(1 + \frac{YZ}{2}\right) V_r + \left(1 + \frac{YZ}{4}\right) ZI_r$$

$$A = D = 1 + \frac{YZ}{2} \qquad B = Z\left(1 + \frac{YZ}{4}\right) \qquad C = Y$$

Short Transmission Line Representation

$$A = D = 1 + \frac{YZ}{2} + \frac{(YZ)^2}{24} + \dots\dots\dots$$

$$B = Z\left[1 + \frac{YZ}{6} + \frac{(YZ)^2}{120} + \dots\dots\dots\right]$$

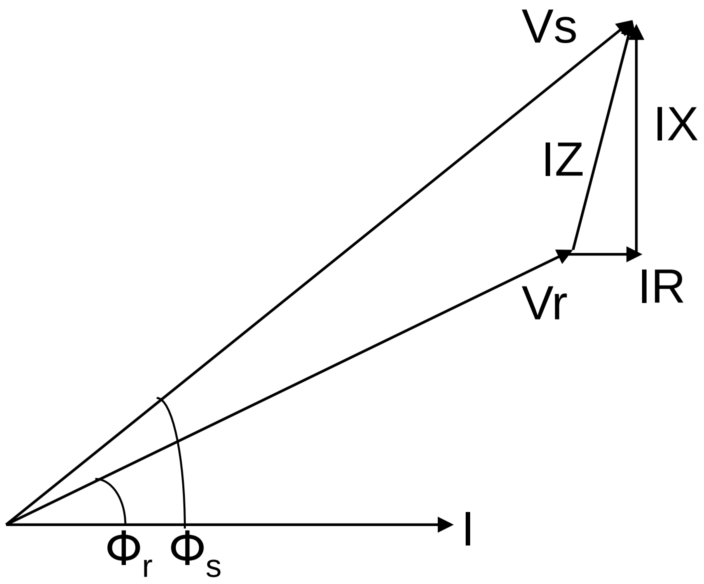
$$C = Y\left[1 + \frac{YZ}{6} + \frac{(YZ)^2}{120} + \dots\dots\dots\right]$$

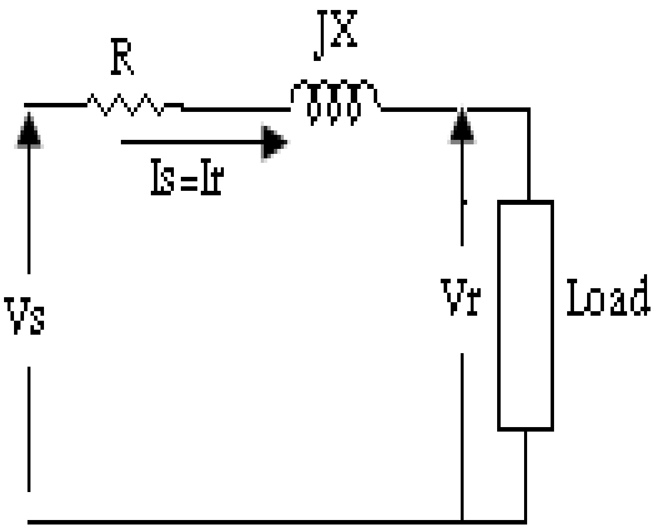
$C = 0$ Because $y = 0$

also, $A = D = 1$, and $\mathrm{B} = \mathrm{Z}$

$$\begin{bmatrix} \mathrm{V_S} \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & \mathrm{Z} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix}$$

Continue





Notes

$$P_r = 3V_r I \cos\varphi_r$$

$$I = I_r = I_S$$

$$P_S = 3V_s I \cos\varphi_s$$

$$\eta = \frac{P_r}{P_s} = \frac{V_r \cos\varphi_r}{V_S \cos\varphi_s}$$

Solved Examples to Evaluate the Different Types of Transmission Lines

Example 1 *A single phase overhead transmission line delivers 1100 kW at 33 kV at 0·8 p.f. lagging. The total resistance and inductive reactance of the line are 10 Ω and 15 Ω respectively. Determine : (i) sending end voltage (ii) sending end power factor and (iii) transmission efficiency.*

Solution.

Load power factor, $\cos\phi_R = 0{\cdot}8$ lagging

Total line impedance, $\vec{Z} = R + j\,X_L = 10 + j\,15$

Receiving end voltage, $V_R = 33\text{ kV} = 33{,}000\text{ V}$

$$\text{Line current, } I = \frac{kW \times 10^3}{V_R \cos\phi_R} = \frac{1100 \times 10^3}{33{,}000 \times 0{\cdot}8} = 41{\cdot}67\text{ A}$$

$$\text{As } \cos\phi_R = 0{\cdot}8 \quad \therefore \quad \sin\phi_R = 0{\cdot}6$$

Performance of Single Phase Short Transmission Lines

$$\vec{V}_R = V_R + j\,0 = 33000 \text{ V}$$

$$\vec{I} = I(\cos\phi_R - j\sin\phi_R)$$

$$= 41{\cdot}67\,(0{\cdot}8 - j\,0{\cdot}6) = 33{\cdot}33 - j\,25$$

(*i*) Sending end voltage, $\vec{V}_S = \vec{V}_R + \vec{I}\,\vec{Z}$

$$= 33{,}000 + (33{\cdot}33 - j\,25{\cdot}0)\,(10 + j\,15)$$

$$= 33{,}000 + 333{\cdot}3 - j250 + j500 + 375$$

$$= 33{,}708{\cdot}3 + j\,250$$

∴ Magnitude of $V_S = \sqrt{(33{,}708\cdot 3)^2 + (250)^2}$ = **33,709 V** (*ii*) Angle between $\vec{V_S}$ and $\vec{V_R}$ is

$$\alpha = \tan^{-1} \frac{250}{33{,}708 \cdot 3} = \tan^{-1} 0{\cdot}0074 = 0{\cdot}42^\circ$$

∴ Sending end power factor angle is

$$\phi_S = \phi_R + \alpha = 36{\cdot}87^0 + 0{\cdot}42^\circ = 37{\cdot}29^\circ$$

∴ Sending end p.f., $\cos \phi_S = \cos 37{\cdot}29^0 =$ **0·7956 lagging**

(*iii*) Line losses $= I^2R = (41{\cdot}67)^2 \times 10 = 17{,}364 \text{ W} = 17{\cdot}364 \text{ kW}$

Output delivered $= 1100 \text{ kW}$

Power sent $= 1100 + 17{\cdot}364 = 1117{\cdot}364 \text{ kW}$

∴ Transmission efficiency $= \dfrac{\text{Power delivered}}{\text{Power sent}} \times 100 = \dfrac{1100}{1117 \cdot 364} \times 100 =$ **98·44%**

Note. V_S and ϕ_S can also be calculated as follows :

$$V_S = V_R + IR\cos\phi_R + IX_L\sin\phi_R \text{ (approximately)}$$

$$= 33{,}000 + 41{\cdot}67 \times 10 \times 0{\cdot}8 + 41{\cdot}67 \times 15 \times 0{\cdot}6$$

$$= 33{,}000 + 333{\cdot}36 + 375{\cdot}03$$

$$= 33708{\cdot}39 \text{ V which is approximately the same as above}$$

$$\cos\phi_S = \frac{V_R\cos\phi_R + IR}{V_S} = \frac{33{,}000 \times 0{\cdot}8 + 41{\cdot}67 \times 10}{33{,}708{\cdot}39} = \frac{26{,}816{\cdot}7}{33{,}708{\cdot}39}$$

$$= 0{\cdot}7958$$

Medium Transmission Lines

Example 2 A (medium) single phase transmission line 100 km long has the following constants :

Resistance/km = 0·25 Ω ; Reactance/km = 0·8 Ω

Susceptance/km = 14×10^{-6} siemen ; Receiving end line voltage = 66,000 V

Assuming that the total capacitance of the line is localised at the receiving end alone, determine (i) the sending end current (ii) the sending end voltage (iii) regulation and (iv) supply power factor. The line is delivering 15,000 kW at 0.8 power factor lagging. Draw the phasor diagram to illustrate your calculations.

Medium Transmission Lines

Total resistance, $R = 0{\cdot}25 \times 100 = 25\ \Omega$

Total reactance, $X_L = 0{\cdot}8 \times 100 = 80\ \Omega$

Total susceptance, $Y = 14 \times 10^{-6} \times 100 = 14 \times 10^{-4}\ S$

Receiving end voltage, $V_R = 66{,}000\ \text{V}$

∴ Load current, $I_R = \dfrac{15{,}000 \times 10^3}{66{,}000 \times 0 \cdot 8} = 284\ \text{A}$

$$\cos\phi_R = 0{\cdot}8\ ; \qquad \sin\phi_R = 0{\cdot}6$$

Taking receiving end voltage as the reference phasor [see Fig.10.10 (*ii*)], we have,

$$\vec{V_R} = V_R + j\,0 = 66{,}000\,V$$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 284\,(0{\cdot}8 - j\,0{\cdot}6) = 227 - j\,170$

Medium Transmission Lines

Capacitive current, $\vec{I_C} = jY \times V_R = j\,14 \times 10^{-4} \times 66000 = j\,92$

(*i*) Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C} = (227 - j\,170) + j\,92$

$$= 227 - j\,78 \qquad \dots(i)$$

Magnitude of $I_S = \sqrt{(227)^2 + (78)^2} =$ **240 A**

(*ii*) Voltage drop $= \vec{I_S}\,\vec{Z} = \vec{I_S}(R + j\,X_L) = (227 - j\,78)\,(\,25 + j\,80)$

$$= 5{,}675 + j\,18,160 - j\,1950 + 6240$$

$$= 11{,}915 + j\,16{,}210$$

Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I_S}\,\vec{Z} = 66{,}000 + 11{,}915 + j\,16{,}210$

$$= 77{,}915 + j\,16{,}210 \qquad \dots(ii)$$

Magnitude of $V_S = \sqrt{(77915)^2 + (16210)^2} =$ **79583V**

Medium Transmission Lines

(*iii*) % Voltage regulation $= \frac{V_S - V_R}{V_R} \times 100 = \frac{79{,}583 - 66{,}000}{66{,}000} \times 100 =$ **20·58%**

(*iv*) Referring to exp. (*i*), phase angle between $\vec{V_R}$ and $\vec{I_S}$ is :

$$\theta_1 = \tan^{-1} - 78/227 = \tan^{-1}(-0{\cdot}3436) = -18{\cdot}96^\circ$$

Referring to exp. (*ii*), phase angle between $\vec{V_R}$ and $\vec{V_S}$ is :

$$\theta_2 = \tan^{-1}\frac{16210}{77915} = \tan^{-1}(0\cdot2036) = 11\cdot50^\circ$$

∴ Supply power factor angle, $\phi_S = 18{\cdot}96^\circ + 11{\cdot}50^\circ = 30{\cdot}46^\circ$

∴ Supply p.f. $= \cos\phi_S = \cos 30{\cdot}46^\circ =$ **0·86 lag**

Long Transmission Lines

Example 3 *A 3-φ transmission line 200 km long has the following constants :*

$$\text{Resistance/phase/km} = 0.16\ \Omega$$

$$\text{Reactance/phase/km} = 0.25\ \Omega$$

$$\text{Shunt admittance/phase/km} = 1.5 \times 10^{-6}\ S$$

Calculate by rigorous method the sending end voltage and current when the line is delivering a load of 20 MW at 0·8 p.f. lagging. The receiving end voltage is kept constant at 110 kV.

Solution :

Total resistance/phase, $R = 0.16 \times 200 = 32\ \Omega$

Total reactance/phase, $X_L = 0.25 \times 200 = 50\ \Omega$

Total shunt admittance/phase, $Y = j\,1.5 \times 10^{-6} \times 200 = 0.0003 \angle 90^\circ$

Series Impedance/phase, $Z = R + j\,X_L = 32 + j\,50 = 59.4 \angle 58^\circ$

Long Transmission Lines

The sending end voltage V_S per phase is given by :

$$V_S = V_R \cosh\sqrt{Y\,Z} + I_R\sqrt{\frac{Z}{Y}}\sinh\sqrt{Z\,Y} \qquad \ldots(i)$$

Now
$$\sqrt{Z\,Y} = \sqrt{59\cdot4\angle 58^\circ \times 0\cdot0003\angle 90^\circ} = 0\cdot133\angle 74^\circ$$

$$Z\,Y = 0\cdot0178\angle 148^\circ$$

$$Z^2Y^2 = 0\cdot00032\angle 296^\circ$$

$$\sqrt{\frac{Z}{Y}} = \sqrt{\frac{59\cdot4\angle 58^\circ}{0\cdot0003\angle 90^\circ}} = 445\angle -16^\circ$$

$$\sqrt{\frac{Y}{Z}} = \sqrt{\frac{0\cdot0003\angle 90^\circ}{59\cdot4\angle 58^\circ}} = 0\cdot00224\angle 16^\circ$$

$\therefore$
$$\cosh\sqrt{Y\,Z} = 1 + \frac{Z\,Y}{2} + \frac{Z^2\,Y^2}{24} \text{ approximately}$$

$$= 1 + \frac{0\cdot0178}{2}\angle 148^\circ + \frac{0\cdot00032}{24}\angle 296^\circ$$

$$= 1 + 0\cdot0089\angle 148^\circ + 0\cdot0000133\angle 296^\circ$$

$$= 1 + 0\cdot0089\,(-0\cdot848 + j\,0\cdot529) + 0\cdot0000133\,(0\cdot438 - j\,0\cdot9)$$

$$= 0\cdot992 + j\,0\cdot00469 = 0\cdot992\angle 0\cdot26^\circ$$

Long Transmission Lines

$$\sinh\sqrt{YZ} = \sqrt{YZ} + \frac{(YZ)^{3/2}}{6} \text{ approximately}$$

$$= 0{\cdot}133 \angle 74° + \frac{0{\cdot}0024 \angle 222°}{6}$$

$$= 0{\cdot}133 \angle 74° + 0{\cdot}0004 \angle 222°$$

$$= 0{\cdot}133\,(0{\cdot}275 + j\,0{\cdot}961) + 0{\cdot}0004\,(-\,0{\cdot}743 - j\,0{\cdot}67)$$

$$= 0{\cdot}0362 + j\,0{\cdot}1275 = 0{\cdot}1325 \angle 74°6'$$

Receiving end voltage per phase is

$$V_R = 110 \times 10^3/\sqrt{3} = 63508 \text{ V}$$

Receiving end current, $$I_R = \frac{20 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0{\cdot}8} = 131\,A$$

Long Transmission Lines

Putting the various values in exp (*i*), we get,

$$V_S = 63508 \times 0.992 \angle\ 0.26^\circ + 131 \times 445 \angle -\ 16^\circ 0' \times 0.1325 \angle\ 74^\circ 6'$$

$$= 63000 \angle\ 0.26^\circ + 7724 \angle\ 58^\circ 6'$$

$$= 63000\,(0.999 + j\,0.0045) + 7724\,(0.5284 + j\,0.8489)$$

$$= 67018 + j\,6840 = 67366 \angle\ 5^\circ 50'\ \text{V}$$

Sending end line-to-line voltage $= 67366 \times \sqrt{3} = 116.67 \times 10^3\ \text{V} =$ **116·67 kV**

The sending end current I_S is given by :

$$I_S = V_R \sqrt{\frac{Y}{Z}} \sinh \sqrt{Y Z} + I_R \cosh \sqrt{Y Z}$$

Putting the various values, we get,

$$I_S = 63508 \times 0.00224 \angle\ 16^\circ \times 0.1325 \angle\ 74^\circ 6' + 131 \times 0.992 \angle\ 0.26^\circ$$

$$= 18.85 \angle\ 90^\circ 6' + 130 \angle\ 0.26^\circ$$

$$= 18.85\,(-0.0017 + j\,0.999) + 130\,(0.999 + j\,0.0045)$$

$$= 129.83 + j\,19.42 = 131.1 \angle\ 8^\circ\ \text{A}$$

∴ Sending end current = **131·1 A**

Circle diagram at no load (Φr=0)

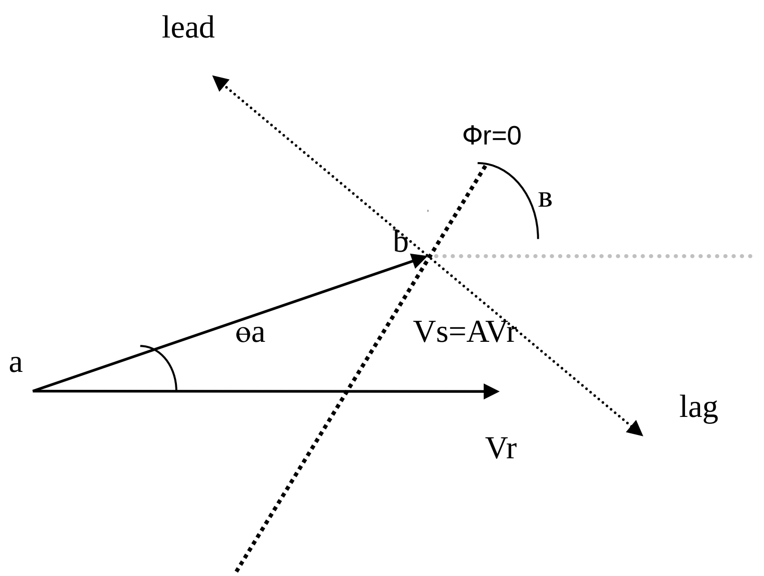
$$I_r = 0 \qquad \phi_r = 0$$

$$V_s\angle\theta = AV_r\angle\theta_a + BI_r\angle\beta \pm \phi_r$$

$$V_s\angle\theta = AV_r\angle\theta_a$$

$$\theta = \theta_a$$

$$\overline{ab} = |V_s| = |AV_r|$$



Circle diagram at Lagging P.f (Φr= -ve value)

$$V_s\angle\theta = AV_r\angle\theta_a + BI_r\angle\beta - \phi_r$$

$$\overline{ac_1} = V_{s1} \qquad \text{at } \frac{1}{4} \text{ load}$$

$$\overline{ac_2} = V_{s2} \qquad \text{at } \frac{1}{2} \text{ load}$$

$$\overline{ac_3} = V_{s3} \qquad \text{at } \frac{3}{4} \text{ load}$$

$$\overline{ac} = V_S \qquad \text{at full load}$$

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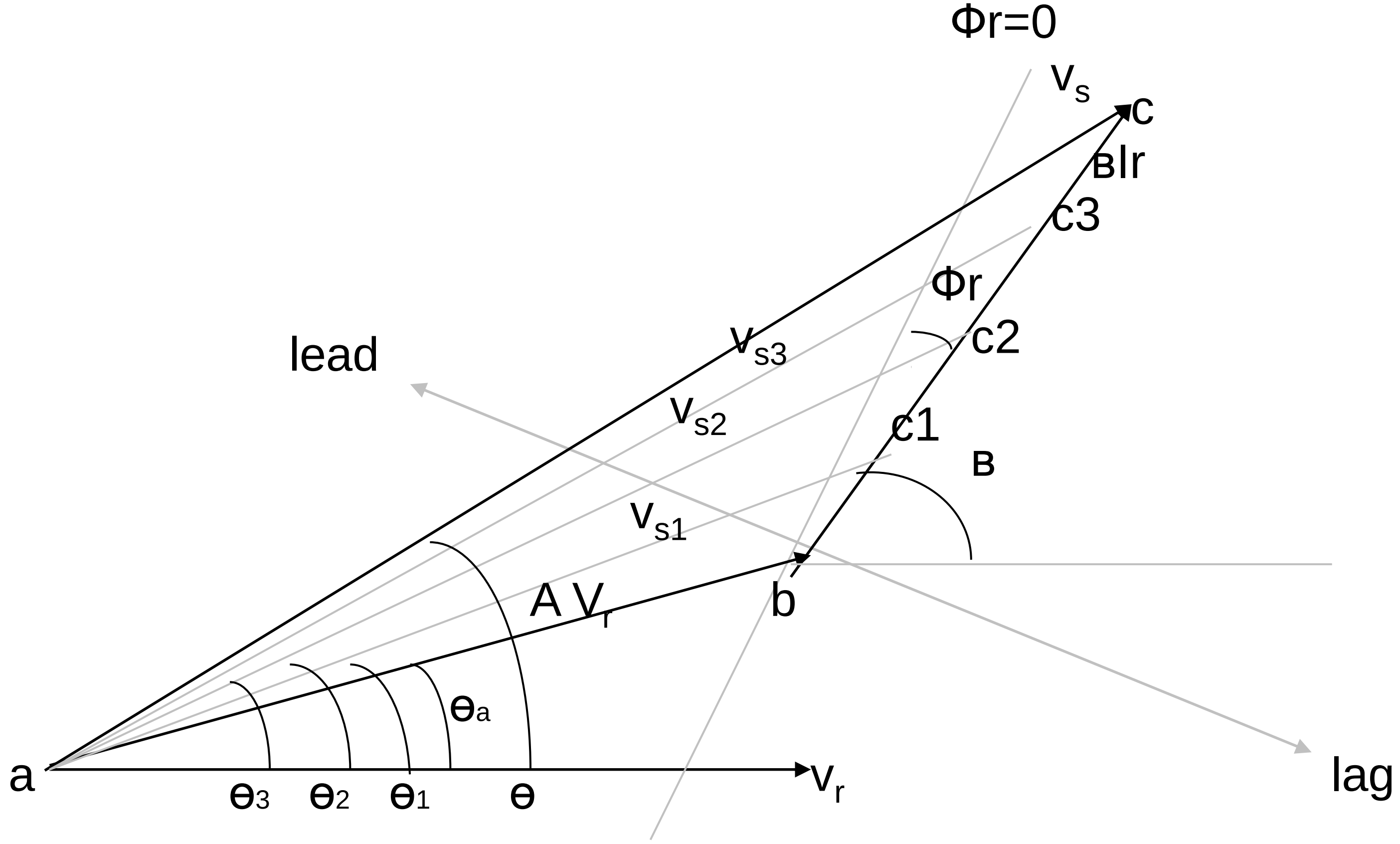

Φr=0
Vs
c
ɞIr
c3
Φr
c2
Vs3
lead
Vs2
c1
ɞ
Vs1
A Vr
b
ϴa
a
ϴ3
ϴ2
ϴ1
ϴ
Vr
lag

Circle diagram at Leading P.f (Φr= +ve value)

$$V_s\angle\theta = AV_r\angle\theta_a + BI_r\angle\beta + \phi_r$$

$$\overline{bc} = BI_r$$

$$\overline{ac} = V_s$$

$$c\widehat{a}d = \theta$$

$$\overline{cD} = BI_r \sin\phi_r$$

$$\overline{bD} = BI_r \cos\phi_r$$

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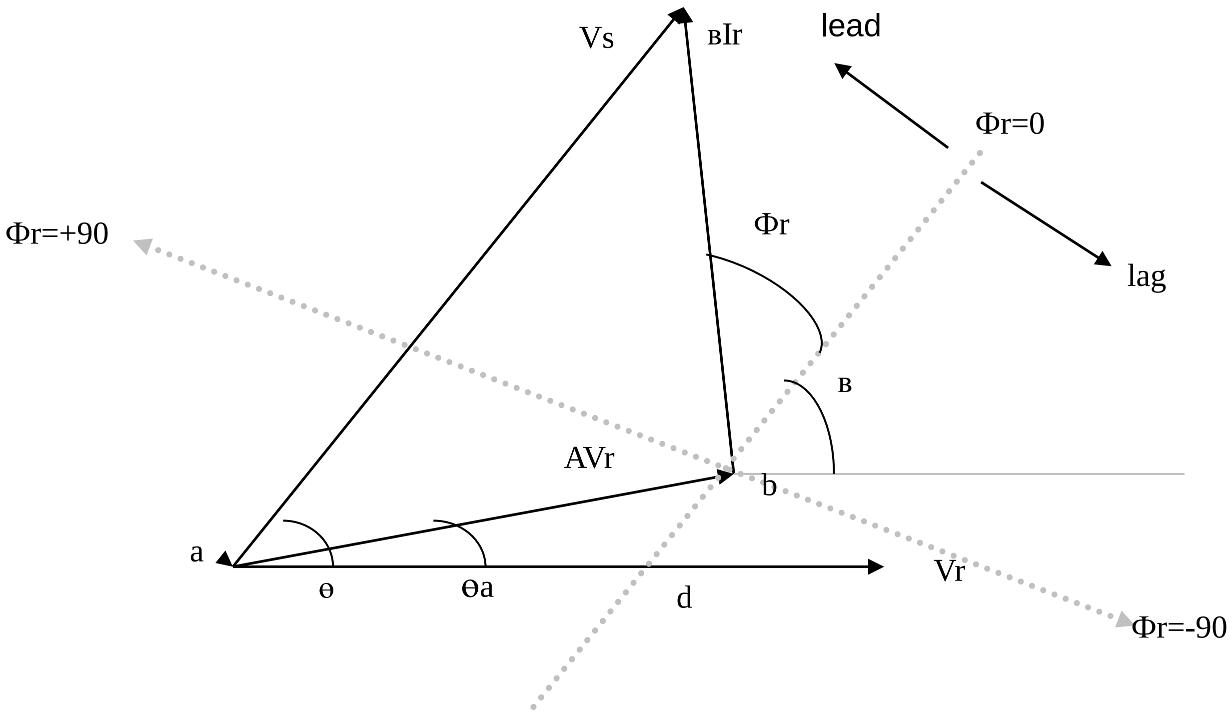
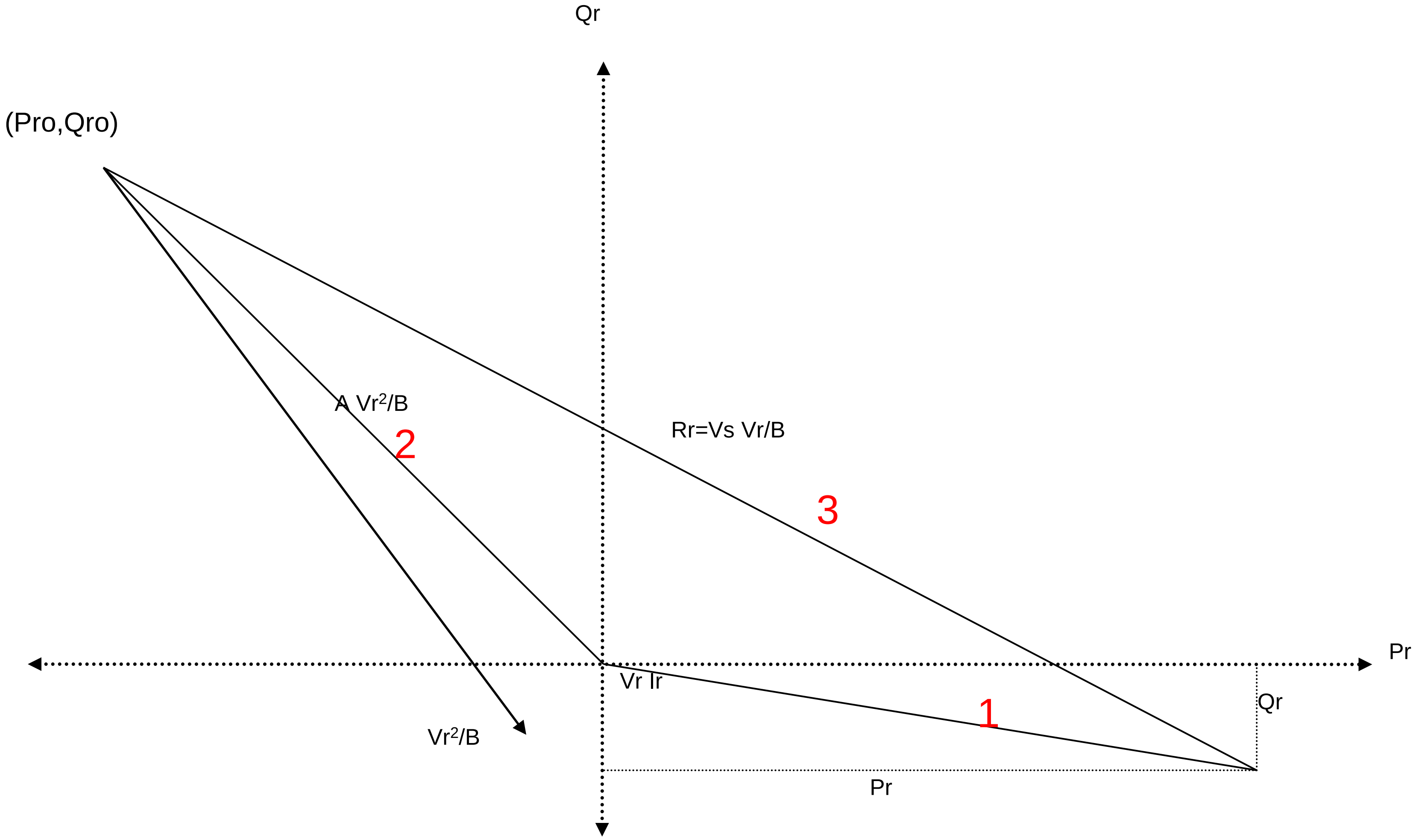


Receiving-End Power Circle Diagram



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$$V_s = AV_r + BI_r \qquad I_r = \frac{V_s}{B} - \frac{AV_r}{B}$$

$$I_r^* = \frac{V_s^*}{B^*} - \frac{A^* V_r^*}{B^*}$$

$$P_r + jQ_r = V_r I_r^*$$

$$P_r + jQ_r = \left(-\frac{A^* V_r^*}{B^*}\right)V_r + \left(\frac{V_s^*}{B^*}\right)V_r$$

$$P_r + jQ_r = -\frac{A^*}{B^*}\left|V_r\right|^2 + \frac{\left|V_r V_s\right|}{B^*}e^{-j\theta}$$

Continue

$$P_{ro} = -\frac{AV_r^2}{B}\cos(\beta - \theta_a)$$

$$Q_{ro} = +\frac{AV_r^2}{B}\sin(\beta - \theta_a)$$

$$P_{ro} = -\frac{AV_r^2}{B}(\cos\beta\cos\theta_a + \sin\beta\sin\theta_a)$$

$$Q_{ro} = \frac{AV_r^2}{B}(\sin\beta\cos\theta_a - \cos\beta\sin\theta_a)$$

Continue

assume,

$$A = a_1 + ja_2 = A\angle\theta_a \qquad , B = b_1 + jb_2 = B\angle\beta$$

$$\sin\theta_a = \frac{a_2}{A} \qquad \cos\theta_a = \frac{a_1}{A}$$

$$\sin\beta = \frac{b_2}{B} \qquad \cos\beta = \frac{b_1}{B}$$

$$Q_{ro} = \frac{+AV_r^2}{B}\left[\frac{b_2 a_1}{AB} - \frac{b_1 a_2}{AB}\right] = \frac{+V_r^2}{B^2}\left[a_1 b_2 - a_2 b_1\right]$$

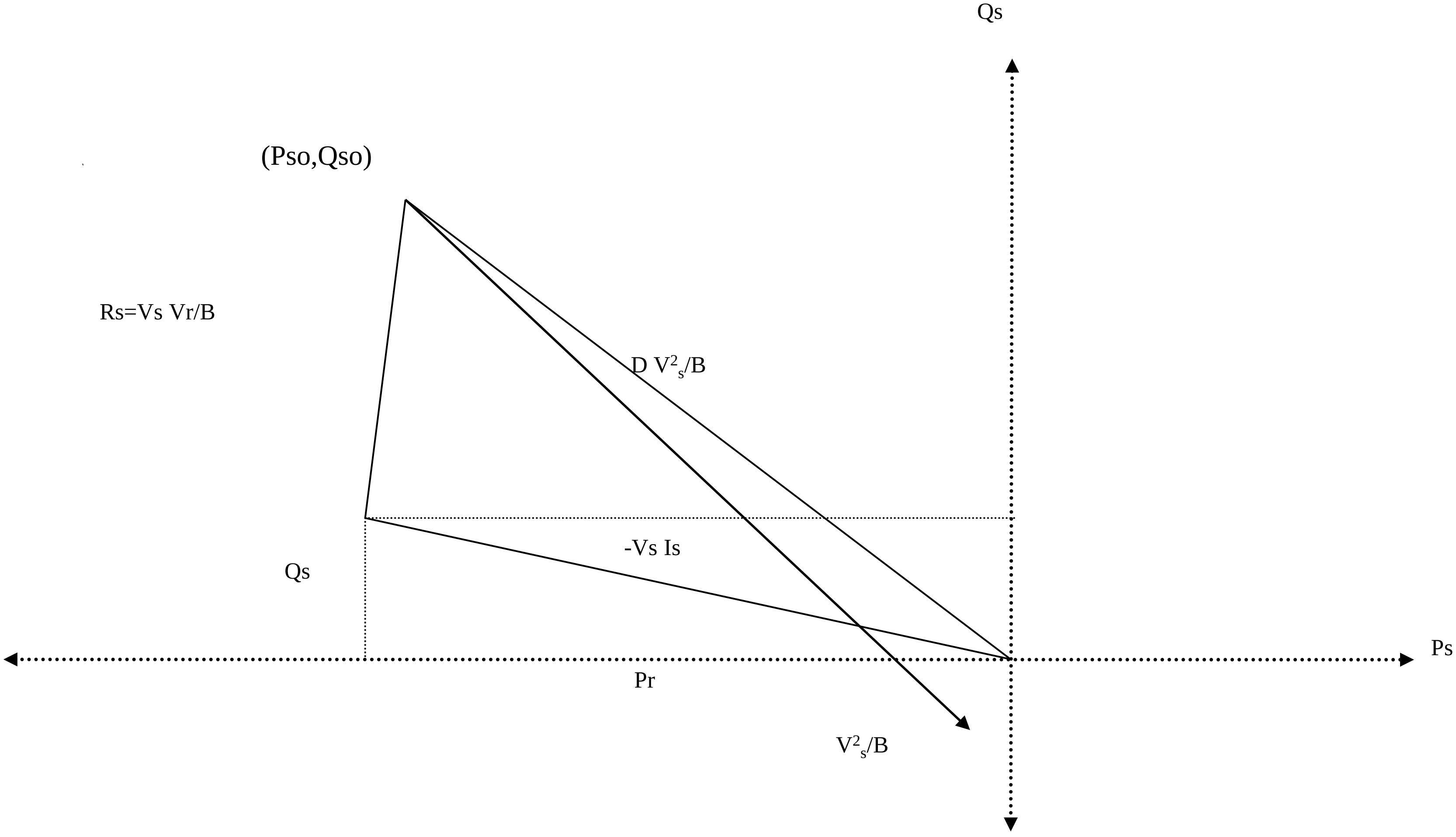
$$P_{ro} = \frac{-AV_r^2}{B}\left[\frac{b_1 a_1}{AB} + \frac{b_2 a_2}{AB}\right] = \frac{-V_r^2}{B^2}\left[a_1 b_1 + a_2 b_2\right]$$

Continue

$$\left(P_r - P_{ro}\right)^2 + \left(Q_r - Q_{ro}\right)^2 = R_r^{\,2}$$

$$\left(P_r + \frac{a_1 b_1 + a_2 b_2}{b_1^{\,2} + b_2^{\,2}} V_r^{\,2}\right)^2 + \left(Q_r - \frac{\left(a_1 b_2 - a_2 b_1\right)}{b_1^{\,2} + b_2^{\,2}} V_r^{\,2}\right)^2 = \frac{V_s^{\,2} V_r^{\,2}}{b_1^{\,2} + b_2^{\,2}}$$

Sending End Power Circle Diagram



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$$DV_s - BI_s = DAV_r - BCV_r + BDI_r - BDI_r$$

$$= (DA - BC) = V_r$$

$$\overline{oa} = \frac{DV_s^2}{B}$$

$$\overline{ab} = -\frac{BI_sV_s}{B} = -I_sV_s$$

$$\overline{ob} = \frac{V_sV_r}{B} = R_s$$

Continue

$$\overline{al} = I_s V_s \cos\phi_s = P_s$$

$$\overline{bl} = V_s I_s \sin\phi_s = Q_s$$

$$P_{so} = \overline{om} = \frac{D V_s^{\,2}}{B}\cos(\beta - \theta_a)$$

$$Q_{so} = \overline{on} = \frac{D V_s^{\,2}}{B}\sin(\beta - \theta_a)$$

Continue

$$V_r = DV_s - BI_s$$

$$= (d_1 + jd_2)V_s - (I_{sp} + jI_{sq})(b_1 + jb_2)$$

$$V_r = (d_1V_s - b_1I_{sp} + b_2I_{sq}) + j(d_2V_s - b_2I_{sp} - b_1I_{sq})$$

$$V_r^2 = (d_1^2 + d_2^2)V_s^2 + (b_1^2 + b_2^2)(I_{SP}^2 + I_{SQ}^2)$$

$$- 2(d_1b_1 + d_2b_2)V_sI_{sp} - 2(d_2b_1 - d_1b_2)V_sI_{sq}$$

Continue

$$\frac{Vr^2}{b1^2+b2^2}=\frac{d1^2+d2^2}{b1^2+b2^2}Vs^2$$

$$+\left[I_{sp}^2-2\frac{(d1b1+d2b2)}{b1^2+b2^2}V_s I_{sp}\right]+\left[I_{sq}^2-2\frac{(d_2b_1-d_1b_2)}{b_1^2+b_2^2}V_s I_{sq}\right]$$

$$\frac{V_r^2 V_s^2}{b_1^2+b_2^2}=\left[I_{sp}V_s-\frac{(d_1b_1+d_2b_2)}{b_1^2+b_2^2}V_s^2\right]^2$$

$$+\left[I_{sq}V_s+\frac{(-d_2b_1+d_1b_2)}{b_1^2+b_2^2}V_s^2\right]^2$$

Continue

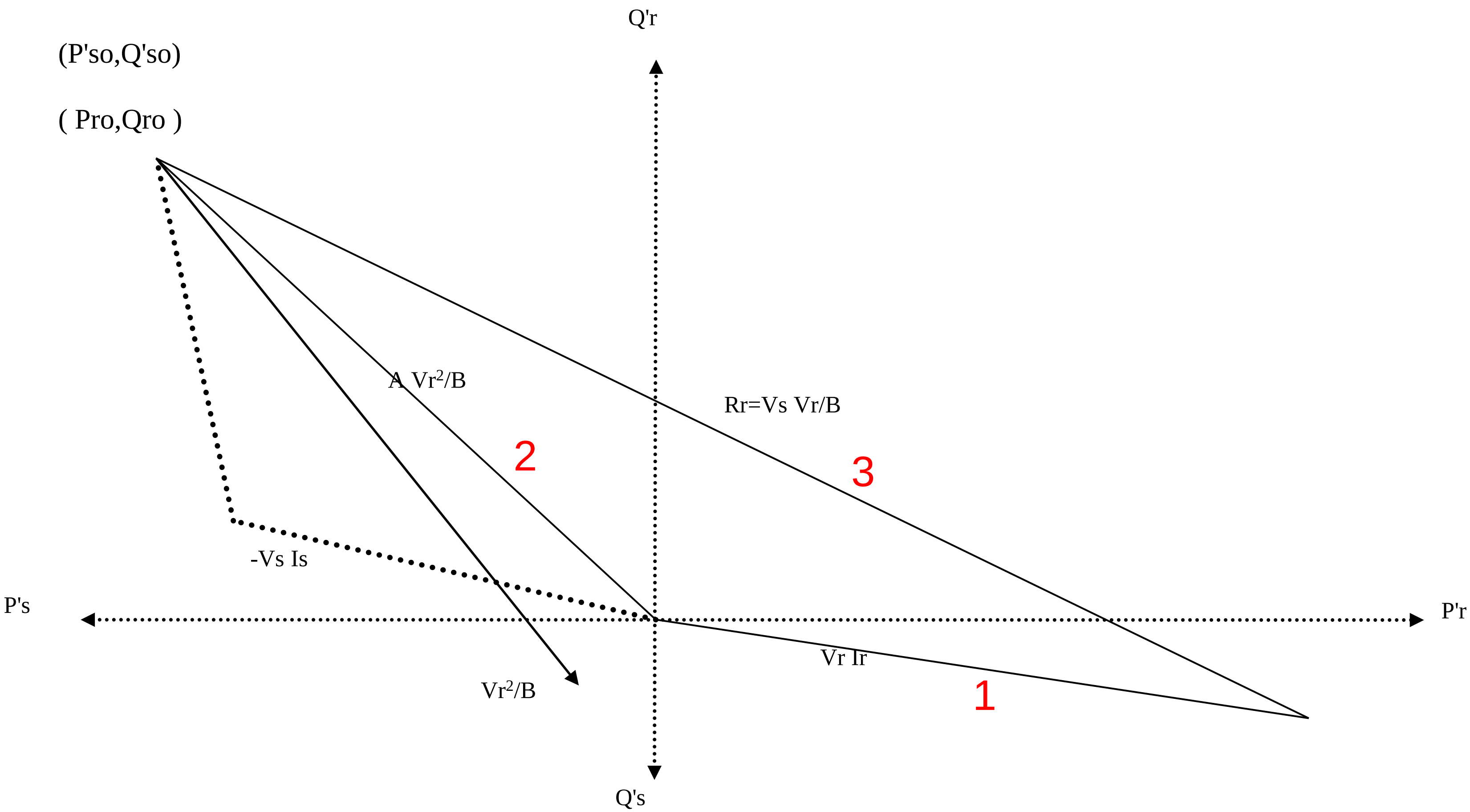
$$R_s^2 = (P_s - P_{so})^2 + (Q_s - Q_{so})^2$$

$$P_{so} = +\frac{(d_1 b_1 + d_2 b_2)}{b_1^2 + b_2^2} V_s^2$$

$$Q_{so} = -\frac{(d_2 b_1 + d_1 b_2)}{b_1^2 + b_2^2} V_s^2$$

$$R_s = \frac{V_s V_r}{B}$$

The Universal Power Circle Diagram



Continue

from R.E. PCD:

$$\left[P_r+\frac{a_1b_1+a_2b_2}{b_1^2+b_2^2}V_r^2\right]^2+\left[Q_r-\frac{(a_1b_2-a_2b_1)}{b_1^2+b_2^2}V_r^2\right]^2=\frac{V_s^2V_r^2}{B^2}$$

from S.E. PCD:

$$\left[P_s-\frac{d_1b_1+d_2b_2}{b_1^2+b_2^2}V_s^2\right]^2+\left[Q_s+\frac{(d_1b_2-d_2b_1)}{b_1^2+b_2^2}V_s^2\right]^2=\frac{V_s^2V_r^2}{B^2}$$

$$\left[P_r\left(\frac{V_b}{V_r}\right)^2+\frac{a_1b_1+a_2b_2}{b_1^2+b_2^2}V_b^2\right]^2+\left[Q_r\left(\frac{V_b}{V_r}\right)^2-\frac{(a_1b_2-a_2b_1)}{b_1^2+b_2^2}V_b^2\right]^2=\frac{V_s^2V_b^4}{V_r^2B^2}$$

Continue

$$\left[P_s\left(\frac{V_b}{V_s}\right)^2 - \frac{a_1b_1 + a_2b_2}{b_1^2 + b_2^2}V_b^2\right]^2 + \left[Q_s\left(\frac{V_b}{V_s}\right)^2 + \frac{\left(a_1b_2 - a_2b_1\right)}{b_1^2 + b_2^2}V_b^2\right]^2$$

$$= \frac{V_b^4 V_r^2}{V_s^2 B^2}$$

$$\left(\bar{P}_r + \bar{P}_{ro}\right)^2 + \left(\bar{Q}_r - \bar{Q}_{ro}\right)^2 = \bar{R}_r^2$$

$$\left(\bar{P}_s - \bar{P}_{so}\right)^2 + \left(\bar{Q}_s + \bar{Q}_{so}\right)^2 = \bar{R}_s^2$$

Continue

It is noted that :

$$\overline{\mathrm{P}}_{\mathrm{ro}} = -\overline{\mathrm{P}}_{\mathrm{so}} = \frac{a_1 b_1 + a_2 b_2}{b_1^{\,2} + b_2^{\,2}} V_b^{\,2}$$

$$\overline{\mathrm{Q}}_{\mathrm{ro}} = -\overline{\mathrm{Q}}_{\mathrm{so}} = \frac{a_1 b_2 - a_2 b_1}{b_1^{\,2} + b_2^{\,2}} V_b^{\,2}$$

$$\overline{R}_r = \frac{V_s V_b^{\,2}}{V_r B} \qquad \overline{R}_s = \frac{V_r V_b^{\,2}}{V_s B}$$

$$\overline{\mathrm{P}}_{\mathrm{r}} = \mathrm{P}_{\mathrm{r}} \left(\frac{\mathrm{V}_{\mathrm{b}}}{\mathrm{V}_{\mathrm{r}}} \right)^2 \qquad \overline{Q}_r = Q_r \left(\frac{V_b}{V_r} \right)^2$$

$$\overline{P}_s = P_s \left(\frac{V_b}{V_s} \right)^2 \qquad \overline{Q}_s = Q_s \left(\frac{V_b}{V_s} \right)^2$$

With Our Best Wishes

Transmission and Distribution of Electrical Power

Course Staff

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